**Exam 1** **Name**________________________________

**Math 131-01** **Calculus I** **February 5, 2018**

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## Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 50 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.

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## Limit Laws

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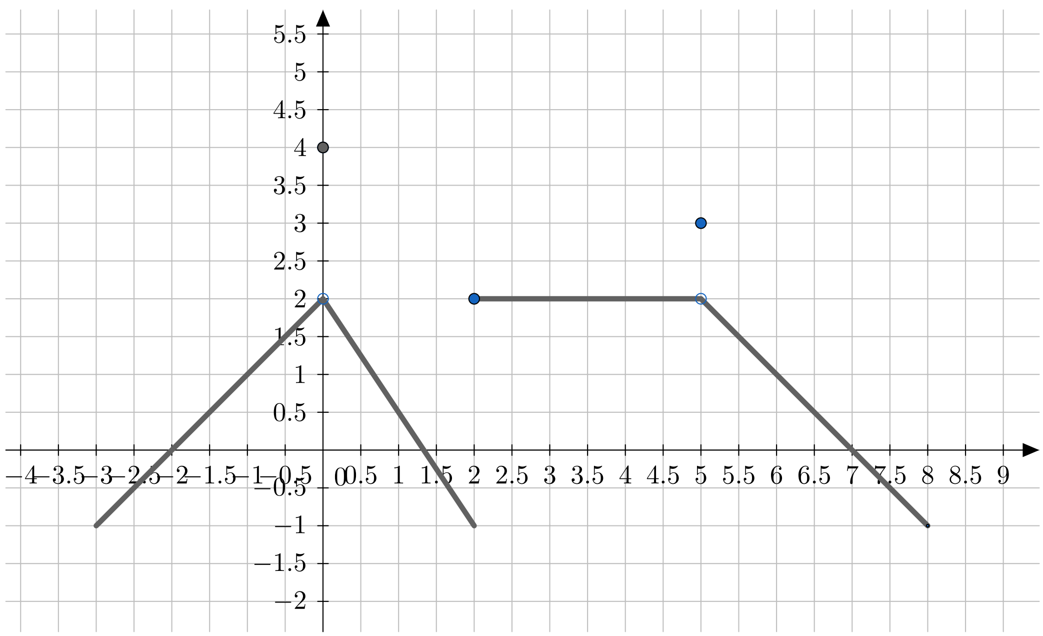
Suppose that $k$ is a constant, and the limits $\lim\limits_{x\to c} f(x)$ and $\lim\limits_{x\to c} g(x)$ exist.

1. Constant Multiple: $\lim\limits_{x\to c} k\, f(x) = k \lim\limits_{x\to c} f(x)$
2. Sum/Difference Rule: $\lim\limits_{x\to c} \Big(f(x) \pm g(x)\Big) = \lim\limits_{x\to c} f(x) \pm \lim\limits_{x\to c} g(x)$
3. Product Rule: $\lim\limits_{x\to c} \Big(f(x)\cdot g(x)\Big) = \lim\limits_{x\to c} f(x) \cdot \lim\limits_{x\to c} g(x)$
4. Quotient Rule: $\lim\limits_{x\to c} \left(\dfrac{f(x)}{g(x)}\right) = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$ if $\lim\limits_{x\to c} g(x) \neq 0$
5. Composition Rule: $\lim\limits_{x\to c} f\big(g(x)\big) = f\big(\lim\limits_{x\to c} g(x)\big)$, if $f$ is continuous at $\lim\limits_{x\to c} g(x)$
6. Cancellation Theorem for Limits: If $\lim\limits_{x\to c} g(x)$ exists and $f$ is a function that is equal to $g$ for all $x$ sufficiently close to $c$, except possibly at $c$ itself, then $\lim\limits_{x\to c} f(x) = \lim\limits_{x\to c} g(x)$.
7. Squeeze Theorem for Limits: If $l(x) \leq f(x) \leq u(x)$ for all $x$ sufficiently close to $c$, but not necessarily at $x = c$, and if $\lim\limits_{x\to c} l(x) = L = \lim\limits_{x\to c} u(x)$, then $\lim\limits_{x\to c} f(x) = L$.
8. Limits Whose Denominators Approach Zero from the Right or the Left:
   (a) If $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)}$ is of the form $\dfrac{1}{0^+}$, then $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = \infty$.
   (b) If $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)}$ is of the form $\dfrac{1}{0^-}$, then $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)} = -\infty$.
9. Limits Whose Denominators Become Infinite Approach Zero:
   (a) If $\lim\limits_{x\to \infty} \dfrac{f(x)}{g(x)}$ is of the form $\dfrac{1}{\infty}$, then $\lim\limits_{x\to \infty} \dfrac{f(x)}{g(x)} = 0$.
   (b) If $\lim\limits_{x\to \infty} \dfrac{f(x)}{g(x)}$ is of the form $\dfrac{1}{-\infty}$, then $\lim\limits_{x\to \infty} \dfrac{f(x)}{g(x)} = 0$.
10. $\lim\limits_{\theta\to 0} \dfrac{\sin(\theta)}{\theta} = 1, \qquad \lim\limits_{\theta\to 0} \dfrac{1-\cos(\theta)}{\theta} = 0, \qquad \lim\limits_{x\to 0} (1+x)^{1/x} = e.$

Similar results hold for limits at infinity and one-sided limits.

1. (6 points) Write an equation of a line through the point $(1, 3)$ and perpendicular to the line through $(1, 3)$ and $(-1, -3)$.

2. (20 points) Use the function $f(x)$ below and limit laws (see cover page, show work using the rules where appropriate) to find the following limits, or explain why they do not exist.



a. $\lim_{x \to 0} f(x)$

b. $\lim_{x \to 2^+} f(x)$

c. $\lim_{x \to 2^-} f(x)$

d. $\lim\limits_{x\to 2} f(x)$

e. $\lim\limits_{x\to 5} \sqrt[3]{4f(x)}$

f. $\lim\limits_{x\to 2^-} \dfrac{2f(x)+3}{f(x)-9}$

3. Evaluate the following limits, if they exist (show your work).

a. (6 points) $\lim\limits_{x\to 3^+} \dfrac{x^2-7x+12}{x^2-5x+6}$

b. (6 points) $\lim\limits_{x\to 0} \dfrac{\frac{1}{4-x}-\frac{1}{4}}{x}$

c. (6 points) $\lim\limits_{x\to 1} \dfrac{\sqrt{2x+7}-3}{x-1}$

d. (6 points) $\lim\limits_{x\to 2} f(x)$, where $f(x) = \begin{cases} x & \text{if } x<2 \\ 4 & \text{if } x=2 \\ 6-2x & \text{if } x>2 \end{cases}$

e. (6 points) $\lim\limits_{x\to 4^-} \dfrac{|x-4|}{x^2-3x-4}$

f. (6 points) $\lim\limits_{x\to \infty} \dfrac{2x^3-2x^2+x}{\sqrt{9x^6+4x^3+x+3}}$

g. (6 points) $\lim_{x\to -2} f(x)$, where $f(x) = \begin{cases} x^2 & \text{if } x \geq -2 \\ 7-x & \text{if } x < -2 \end{cases}$

4. (6 points) If $f(x) = \sqrt{2x^4 + 25x + 9}$, use the Intermediate Value Theorem to show that there is a number $c$ such that $f(c) = 5$ on $[0, 1]$.

5. (8 points) Let

$$f(x) = \begin{cases} x^2 - b & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ ax + 4 & \text{if } x > 1 \end{cases}$$

Find values of $a$ and $b$ such that $f$ is continuous at $x = 2$. Use the definition of continuity in your answer.

6. (10 points) Sketch a graph of a function $f$ that satisfies all the following conditions.

$$\begin{array}{lll} \lim_{x\to 1} f(x) = -\infty & f(1) = 0 & f(3) = 2 \\ \lim_{x\to 3^-} f(x) = 3 & \lim_{x\to 3^+} f(x) = 1 & \\ \lim_{x\to \infty} f(x) = 2 & \lim_{x\to -\infty} f(x) = 2 & \end{array}$$

7. (8 points) To be done when exam is graded and returned. On a separate sheet of paper, correct ALL missed problems if your score is less than 85 points. Then take the corrections and exam to the OSL, a tutor will check your work. They will also explain questions you are having difficulty correcting.

| Question | Points | Score |
|---|---|---|
| 1 | 6 | |
| 2 | 20 | |
| 3 | 42 | |
| 4 | 6 | |
| 5 | 8 | |
| 6 | 10 | |
| 7 | 8 | |
| Total: | 100 | |