## Name <br> Calculus I <br> September 10, 2018

## Guidelines

- Calculators are not allowed.
- Read the questions carefully. You have 50 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln (2)$, unless they simplify further like $\sqrt{9}=3$ or $\cos (3 \pi / 4)=-\sqrt{2} / 2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write "see other side" and continue working on the back of the same page.

OSL Tutors,
Thank you very much for the thought and care you use when checking solutions for test corrections.

Signature: $\qquad$
Print: $\qquad$
Date: $\qquad$

## Limit Laws

Suppose that $k$ is a constant, and the limits $\lim _{x \rightarrow c} f(x)$ and $\lim _{x \rightarrow c} g(x)$ exist.

1. Constant Multiple: $\lim _{x \rightarrow c} k f(x)=k \lim _{x \rightarrow c} f(x)$
2. Sum/Difference Rule: $\lim _{x \rightarrow c}(f(x) \pm g(x))=\lim _{x \rightarrow c} f(x) \pm \lim _{x \rightarrow c} g(x)$
3. Product Rule: $\lim _{x \rightarrow c}(f(x) \cdot g(x))=\lim _{x \rightarrow c} f(x) \cdot \lim _{x \rightarrow c} g(x)$
4. Quotient Rule: $\lim _{x \rightarrow c}\left(\frac{f(x)}{g(x)}\right)=\frac{\lim _{x \rightarrow c} f(x)}{\lim _{x \rightarrow c} g(x)}$ if $\lim _{x \rightarrow c} g(x) \neq 0$
5. Composition Rule: $\lim _{x \rightarrow c} f(g(x))=f\left(\lim _{x \rightarrow c} g(x)\right)$, if $f$ is continuous at $\lim _{x \rightarrow c} g(x)$
6. Cancellation Theorem for Limits: If $\lim _{x \rightarrow c} g(x)$ exists and $f$ is a function that is equal to $g$ for all $x$ sufficiently close to $c$, except possibly at $c$ itself, then $\lim _{x \rightarrow c} f(x)=\lim _{x \rightarrow c} g(x)$.
7. Squeeze Theorem for Limits: If $l(x) \leq f(x) \leq u(x)$ for all $x$ sufficiently close to $c$, but not necessarily at $x=c$, and if $\lim _{x \rightarrow c} l(x)=L=\lim _{x \rightarrow c} u(x)$, then $\lim _{x \rightarrow c} f(x)=L$.
8. Limits Whose Denominators Approach Zero from the Right or the Left:
(a) If $\lim _{x \rightarrow c} \frac{f(x)}{g(x)}$ is of the form $\frac{1}{0^{+}}$, then $\lim _{x \rightarrow c} \frac{f(x)}{g(x)}=\infty$.
(b) If $\lim _{x \rightarrow c} \frac{f(x)}{g(x)}$ is of the form $\frac{1}{0^{-}}$, then $\lim _{x \rightarrow c} \frac{f(x)}{g(x)}=-\infty$.
9. Limits Whose Denominators Become Infinite Approach Zero:
(a) If $\lim _{x \rightarrow \infty} \frac{f(x)}{g(x)}$ is of the form $\frac{1}{\infty}$, then $\lim _{x \rightarrow \infty} \frac{f(x)}{g(x)}=0$.
(b) If $\lim _{x \rightarrow \infty} \frac{f(x)}{g(x)}$ is of the form $\frac{1}{-\infty}$, then $\lim _{x \rightarrow \infty} \frac{f(x)}{g(x)}=0$.
10. $\lim _{\theta \rightarrow 0} \frac{\sin (\theta)}{\theta}=1, \quad \lim _{\theta \rightarrow 0} \frac{1-\cos (\theta)}{\theta}=0, \quad \lim _{x \rightarrow 0}(1+x)^{1 / x}=e$.

Similar results hold for limits at infinity and one-sided limits.

1. (8 points) Complete test corrections: If your score is below 85, on a separate page neatly correct any questions where points were deducted. Take your test corrections to the Office of Student Learning, Speare 110. A tutor will go over your corrections with you, verifying that the answers are now correct. The signed corrections will be returned to your instructor.
2. Use the function $f(x)$ below and limit laws (see previous page, show work using the rules where appropriate) to find the following limits, or explain why they do not exist.

a. (2 points) $\lim _{x \rightarrow-1} f(x)=$ $\qquad$
b. (2 points) $f(-1)=$ $\qquad$
c. (2 points) $\lim _{x \rightarrow 6^{+}} f(x)=$ $\qquad$
d. (2 points) $\lim _{x \rightarrow 4^{-}} f(x)=$ $\qquad$
e. $(4$ points $) \lim _{x \rightarrow 1}(2 f(x)+3)=$
f. (4 points) $\lim _{x \rightarrow 3} \sqrt{8 f(x)}=$
g. (4 points) State the equation of any vertical asymptote on $[-3,8]$.
3. Evaluate the following limits, if they exist (show your work).
a. (6 points) $\lim _{x \rightarrow 5} \frac{x^{2}-6 x+5}{x-5}$
b. (6 points) $\lim _{x \rightarrow 3} \frac{\frac{1}{x}-\frac{1}{3}}{x-3}$
c. (6 points) $\lim _{x \rightarrow 4} f(x)$ where $4 x-9 \leq f(x) \leq x^{2}-4 x+7$ for $x \geq 0$
d. (6 points) $\lim _{x \rightarrow 1} f(x)$, where $f(x)=\left\{\begin{array}{cl}x^{2}+1 & \text { if } x<1 \\ (x-2)^{2} & \text { if } x \geq 1\end{array}\right.$
e. (6 points) $\lim _{x \rightarrow 4^{+}} \frac{4-x}{|4-x|}$
f. (6 points) $\lim _{t \rightarrow \infty} \frac{t-t \sqrt{t}}{2 t^{3 / 2}+3 t-5}$
g. (6 points) $\lim _{r \rightarrow 9} \frac{\sqrt{r}}{(r-9)^{4}}$
4. (6 points) If $f(x)=x^{5}-x^{3}+3 x-5$, use the Intermediate Value Theorem to show that $f(x)$ has a root somewhere in the interval [1,2].
5. (8 points) Let

$$
f(x)=\left\{\begin{array}{cc}
3 x+b & \text { if } x<2 \\
4 & \text { if } x=2 \\
a x^{2}+2 & \text { if } x>2
\end{array}\right.
$$

Find values of $a$ and $b$ such that $f$ is continuous at $x=2$. Use the definition of continuity in your answer.
6. (8 points) Find the horizontal and vertical asymptotes for the curve $y=\frac{2 x^{2}+1}{3 x^{2}+2 x-1}$.
7. (8 points) Sketch a graph of a function $f$ that satisfies all the following conditions.

$$
\begin{array}{lll}
\lim _{x \rightarrow-3} f(x)=\infty & \lim _{x \rightarrow 3^{-}} f(x)=-\infty & \lim _{x \rightarrow 3^{+}} f(x)=2 \\
\lim _{x \rightarrow \infty} f(x)=0 & \lim _{x \rightarrow-\infty} f(x)=-2 & \\
f \text { is continuous from right at } 3 & &
\end{array}
$$

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points: | 8 | 20 | 42 | 6 | 8 | 8 | 8 | 100 |
| Score: |  |  |  |  |  |  |  |  |

