

Guidelines

- **Calculators are not allowed.**
- Read the questions carefully. You have 50 minutes; use your time wisely.
- You may leave your answers in symbolic form, like $\sqrt{3}$ or $\ln(2)$, unless they simplify further like $\sqrt{9} = 3$ or $\cos(3\pi/4) = -\sqrt{2}/2$.
- Put a box around your final answers when relevant.
- Show all steps in your solutions and make your reasoning clear. Answers with no explanation will not receive full credit, even when correct.
- Use the space provided. If necessary, write - See other Side - and continue working on the back of the same page.

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1. (8 points) To be completed once exams are graded and returned. Please correct any problem with points deducted. All corrections should be completed neatly on a separate sheet of paper. Once you have finished your corrections, take your exam and corrections to the Office of Student Learning (OSL), and a tutor will check your answers and sign below. The checked solutions should be given to your instructor.

Signature: _____

Print Name: _____

Date: _____

Question	Points	Score
1	8	
2	26	
3	6	
4	6	
5	6	
6	8	
7	8	
8	12	
9	10	
10	10	
Total:	100	

$$y = \arctan x \Leftrightarrow \tan y = x$$

$$\cos^2 x + \sin^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$

$$\sec^2 y \cdot y' = 1$$

$$y' = \frac{1}{\sec^2 y}$$

$$= \frac{1}{1 + \tan^2 y}$$

$$= \frac{1}{1 + x^2}$$

$$u = g(x)$$

$$\frac{d}{dx}(u^p) = p u^{p-1} \cdot u'$$

2. Find $\frac{dy}{dx}$ for each of the following: (Do not simplify)

a. (2 points) $y = x^3 - 3(x^2 + \pi^2)$
 $= x^3 - 3x^2 + 3\pi^2$

b. (2 points) $y = \ln 5$

c. (2 points) $y = e^{2x+3}$

$$\frac{d}{dx}(e^u) = e^u \cdot u'$$

d. (2 points) $y = \sqrt{x^2 + 3x}$

$$y = u^{1/2} \quad y' = \frac{1}{2} u^{-1/2} \cdot u'$$

e. (2 points) $y = \tan(\pi x)$

$$y = \tan u \quad \frac{dy}{dx} = \sec^2 u \cdot u'$$

f. (2 points) $y = \sin(7x)$

$$y' = u' \cdot \cos u$$

g. (2 points) $y = 2^{x^2} = e^{u \ln b} = e^{\underbrace{u \ln 2}_{2 \times \ln 2 \cdot 2^x}}$

h. (2 points) $y = \frac{1}{7x^2 + 3x} = (7x^2 + 3x)^{-1}$

a. $y' = 3x^2 - 6x$

b. $y' = 0$

c. $y' = e^{2x+3} \cdot 2$

d. $y' = \frac{1}{2} (x^2 + 3x)^{-1/2} \cdot (2x + 3)$

e. $y' = \sec^2(\pi x) \cdot \pi$

f. $y' = 7 \cos(7x)$

g. $y' = e^{x^2 \ln 2} \cdot (2 \ln 2)$

h. $-1(7x^2 + 3x)^{-2}(14x + 3)$

i. (2 points) $y = \log_2(4x) = \frac{\ln 4x}{\ln 2} = \frac{1}{\ln 2} \cdot \ln(4x)$

i. $y' = \frac{1}{\ln 2} \cdot \frac{1}{4x} \cdot 4$

j. (2 points) $y = \arcsin \sqrt{x}$

j. $\frac{1}{\sqrt{1-x}} \cdot \frac{1}{2} x^{-1/2}$

k. (2 points) $y = \cos^3(1 - 3x) = u^3$

k. $y' = 3\cos^2(1-3x)(-\sin(1-3x) \cdot (-3))$

$u = \cos(1-3x)$

$w = 1-3x$

l. (2 points) $y = \arctan(9x)$

$$u' \cdot \frac{1}{1+u^2}$$

m. (2 points) $y = \ln(x^{5/2} - 3)$

l. $\frac{9 \cdot \frac{1}{1+(9x)^2}}{x^{5/2}-3} \cdot \frac{5}{2} x^{3/2}$

3. (6 points) Find dy/dx for $y = x^2 \sin^2(2x^2)$

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(x^2) \sin^2(2x^2) + x^2 \cdot \frac{d}{dx}(\sin^2(2x^2)) \\ &= 2x \sin^2(2x^2) + x^2(2\sin(2x^2) \cdot \cos(2x^2) \cdot 4x) \end{aligned}$$

$(\text{low} \cdot \text{high} - \text{high} \cdot \text{low}) / \text{low} \cdot \text{low}$

4. (6 points) Find dy/dx for $y = \frac{13x}{(2\sqrt{x} + 2)^2} = 13x(2\sqrt{x} + 2)^{-2}$

$$y' = \frac{\frac{d}{dx}(13x)(2\sqrt{x} + 2)^2 - 13x \frac{d}{dx}(2\sqrt{x} + 2)^2}{(2\sqrt{x} + 2)^4}$$

$$y' = \frac{13(2\sqrt{x} + 2)^2 - (13x)2(2\sqrt{x} + 2) \cdot x^{-1/2}}{(2\sqrt{x} + 2)^4}$$

5. (6 points) Find dy/dx for $y = 3x + \frac{9}{x}$ using the definition of derivative.

$$\begin{aligned} y' &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\left(3(x+h) + \frac{9}{x+h}\right) - \left(3x + \frac{9}{x}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x+3h-3x+\frac{9}{x+h}-\frac{9}{x}}{h} = \lim_{h \rightarrow 0} \left(\frac{3h}{h} + \frac{1}{h} \left(\frac{9}{x+h} - \frac{9}{x} \right) \right) \xrightarrow{\left(\frac{x+h}{x+h}\right)} \\ &= 3 + \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{9}{x(x+h)} - \frac{9(x+h)}{x(x+h)} \right] = 3 + \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{9x-9x-9h}{x(x+h)} \right] \\ &\quad = 3 + \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{-9h}{x(x+h)} \right] = 3 - \frac{9}{x^2} \end{aligned}$$

6. (8 points) Find dy/dx for $x^3 + 4xy - 3y^{4/3} = 2x$.

$$3x^2 + 4y + 4x \frac{dy}{dx} - 4y^{1/3} \frac{dy}{dx} = 2$$

$$\frac{dy}{dx} (4x - 4y^{1/3}) = 2 - 3x^2 - 4y$$

$$\frac{dy}{dx} = \frac{2 - 3x^2 - 4y}{4x - 4y^{1/3}}$$

$$b^x = e^{\ln b^x} = e^{x \ln b}$$

7. (8 points) Find dy/dx for $y = (1 + 3x^2)^{6x}$.

$$\ln y = \ln(1 + 3x^2)^{6x}$$

$$\ln y = 6x \ln(1 + 3x^2)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = 6 \cdot \ln(1 + 3x^2) + 6x \cdot \frac{1}{1 + 3x^2} \cdot 6x$$

$$\frac{dy}{dx} = y \left[6 \ln(1 + 3x^2) + \frac{36x^2}{1 + 3x^2} \right]$$

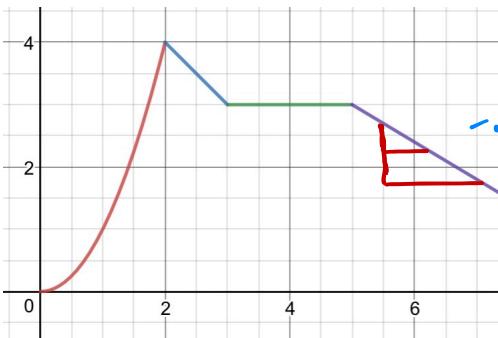
OR

$$y = e^{6x \cdot \ln(1 + 3x^2)}$$

$$y' = e^{6x \ln(1 + 3x^2)}$$

$$= \left[6 \ln(1 + 3x^2) + 6x \cdot \frac{1}{1 + 3x^2} \cdot 6x \right]$$

8. Given the position function, $s = f(t)$, where $0 \leq t \leq 10$, seen below, answer the following questions:



-slope of line

$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{3.5}$$

$$= \frac{-2}{7/2} = -\frac{4}{7}$$

- a. (3 points) $\lim_{h \rightarrow 0} \frac{f(6+h) - f(6)}{h} = f'(6)$ a. $= -\frac{4}{7}$
- b. (3 points) Where is $\frac{ds}{dt} = 0$? b. $3 < x < 5$
- c. (3 points) Where is $f'(t) > 0$? c. $0 < x < 2$
- d. (3 points) Where is $f'(t)$ undefined? d. $x = 2, 3, 5, (0)$

9. (10 points) Find an equation of the line tangent to the curve $y = \tan x$ at $x = -\pi/4$.

$$y - y_0 = m(x - x_0)$$

$$y + 1 = 2(x + \pi/4)$$

$$\left. \begin{array}{l} m = y' \\ y' = \sec^2 x \\ m = \sec^2(-\pi/4) = \left(\frac{2}{\sqrt{2}}\right)^2 = 2 \end{array} \right\} \begin{array}{l} x_0 = -\pi/4 \\ y_0 = \tan(-\pi/4) = -1 \end{array}$$

- ~~10~~ (10 points) The length ℓ of a rectangle is decreasing at a rate of 2 cm/sec while the width w is increasing at a rate of 2 cm/sec. When $\ell = 12$ cm and $w = 5$ cm, find the rate of change of the length of the diagonals of the rectangle.