Ten's Complement Arithmetic

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Many mechanical adding machines, such as the Felt and Tarrant Comptometer and the Hoffritz Adder shown in Figure 1 have somewhat mysterious numerical labels. Next to the digit 4, a 5 appears, next to the digit 3, a 6 appears, and so on. By using these mysterious labels it is possible to perform subtraction with a machine that would normally only be able to perform addition. The system of arithmetic that makes this possible is called ten's complement arithmetic [1].

The Hoffritz adder that we will use in our examples has seven digits. On this machine, 9999999+1 rolls over to 0 instead of 10000000. This behavior is the key to the ten's complement system. If we want to subtract $x - y$, we can instead compute

$$x - y = x + (10000000 - y) - 10000000$$

The number $10000000 - y$ is called the ten's complement of y. Because of the way in which the adder rolls over to 0, it is not actually necessary to subtract off the 10000000.

The tricky part is finding the ten's complement of y. To do this, we write

$$10000000 - y = (9999999 - y) + 1.$$

The number $9999999 - y$ is called the nines' complement of y. Computing $9999999 - y$ is easy. Simply replace easy digit in the number y with nine minus the digit. These are exactly the alternate labels shown in Figure 1.

So, the procedure for subtracting $x - y$ is as follows. Enter x into the adder. Then add the nines' complement of y. Finally, add 1.

For example, suppose that we want to compute $57 - 78$. We start by clearing the adder and then add 57. The nines' complement of 78 is 9999921. After adding 9999921 and adding one more, we get 9999979. This is the ten's complement representation of a negative number. To convert back to the conventional notation, we take the nines complement and get 0000020. Then we add 1 to get 0000021. Thus $57 - 78 = -21$.

In using this scheme to add and subtract numbers it is important to distinguish between positive and negative numbers. By convention, any number in the adder whose leading digit is 0, 1, 2, 3, or 4 represents a positive number, and any number that begins with a 5, 6, 7, 8, or 9 represents a negative number. The number -1 is represented as 9999999. The largest positive number that can be represented on the Hoffritz adder is +4999999. Surprisingly, the most negative number is -5000000, which is represented as 5000000.

Although some early electronic computers used decimal arithmetic and the ten's complement system, most modern computers use binary arithmetic. In binary arithmetic, a similar complementary arithmetic scheme can be used. Rather than ten's complement arithmetic, we now have two's complement arithmetic.

References

[1] Knuth, D. E. *The Art of Computer Programming, Volume 2: Seminumerical Algorithms, 3rd ed.* New York, Addison–Wesley, 1997.

Figure 1: Nines' Complement Labels on the Hoffritz Adder.